Interferometric Measurement of Workpiece Flatness in Ultra-Precision Flycutting

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Abstract

Interferometers can quickly and accurately measure the flatness of workpieces fabricated by manufacturing techniques such as ultra-precision flycutting. The measurement results provide information that may be used to optimize the manufacturing process conditions. In this article we introduce the use of phase measuring interferometers to examine the effects of structural dynamics on the surface figure (flatness) of flycut parts. A model is presented that incorporates the structural dynamics and the loading that results from intermittent contact of a rotating diamond cutting tool with the workpiece. The predicted surface figure is shown to accurately match the interferometer measurements obtained in cutting trials. The model is then used to predict workpiece flatness under a variety of cutting conditions. The results indicate that workpiece flatness is highly dependent on the relationship between the spindle speed, the dominant resonant frequency within the structure, and the swept angle of the interrupted cut. The results provide insight into how interferometry may be used to optimize ultra-precision flycutting operations.

Keywords: Phase measuring interferometry, workpiece inspection, manufacturing process optimization

Introduction

Ultra-precision flycutting, a single point cutting process that generates optical quality surface finishes on flat (plano) workpieces, is characterized by depths of cut ranging from roughing cuts of 25 or 50 micrometers to one micrometer or less for finishing cuts. Such work is done on diamond turning lathes with high speed air bearing spindles. Unlike turning operations, which use a stationary cutting tool that is slowly fed past a rotating workpiece, flycutting operations use a rotating tool that is slowly fed past a stationary workpiece. This arrangement allows flat surfaces to be machined on oddly shaped workpieces without the associated problems of balancing and fixturing that would occur in conventional turning (Chaloux, 1984). An excellent example of ultra-precision flycutting is the production of KDP (potassium dihydrogen phosphate) crystals for the National Ignition Facility laser fusion experiments (Lahaye, 1999 and Namba, 1998).

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Contact between the rotating tool and stationary workpiece is inherently intermittent and the abrupt cycling of the cutting force leads to vibration and workpiece surface figure errors (out-of-flatness). Perhaps the most convenient means of quantifying the flatness of a workpiece is the phase measuring interferometer which offers the benefit of high resolution and accuracy as well as simultaneous measurement of the entire workpiece surface. The interferometer will quickly resolve the undulations in the workpiece surface left behind by the vibrating tool. As will be shown, the workpiece surface figure is sensitive to the relationship between the duty cycle of the cutting force, the dominant resonant frequencies of the structure, and the spindle speed. Using a combination of modeling and interferometer measurements, this paper will show why workpiece flatness is worst when the dominant structural resonance occurs at integer multiples of the spindle speed.

The flycutting system analysis is derived from the rich literature on regenerative chatter. The input to the flycutting system is the intermittent cutting force, which is predicted by a model used extensively in previous work on milling and turning (Tlusty, 1985). Lucca and Rhorer have looked at the specific circumstances of precision cutting processes to characterize the relationship between uncut chip area and cutting force in single point diamond turnable materials such as the aluminum used in this study (Lucca, 1991 and Lucca, 1994). The intermittent contact of the tool and workpiece leads to excitation energy over a range of frequencies occurring at integer multiples of the spindle speed.

This cutting force model drives a lumped parameter model of the machine tool structure based on one or multiple degrees of freedom. For example Tlusty, Hannah and Tobias, and others use single degree of freedom structural models in their explorations of regenerative chatter (Tlusty, 1978, 1980, and 1981 and Hanna, 1974). It is important to note that the cutting force itself is dependent on the flycutting vibration because the force is proportional to the instantaneous area of the uncut chip. Therefore, any vibration in the structural loop leads to fluctuations in the cutting force that are fed back into the structure. The regenerative chatter literature also describes a second situation in which undulations on the surface generated during previous passes of the tool over the workpiece lead to fluctuations in the cutting force and in some cases, instability. This second regenerative effect is both small and stable for most practical flycutting situations and does not appear in the interferometer measurement results of this study. However, the structural dynamics still play the dominant role in the waviness of the finished workpiece.

**Workpiece Flatness Analysis**

The surface of a flycut workpiece accurately records any vibration of the tool-workpiece structural loop, including motion caused by the intermittent cutting force. A flycut workpiece may be inspected at the micrometer level using a scanning white light interferometer to look at the individual passes of the diamond
tool over the workpiece. Tool wear and surface finish are investigated at this size scale. At the macro-level, a large aperture interferometer will show the overall surface figure errors (flatness errors) resulting from compliance and the dynamics of the tool-to-workpiece structural loop. The flatness is also influenced by the workpiece material, the geometry of the workpiece, and the cutting parameters such as depth of cut, feed rate, and spindle speed. In this section we present a flycutting model that will predict the flatness to be compared to the actual macro-level interferometer measurements.

As mentioned above, the key dynamics are modeled with sufficient accuracy using lumped parameter elements and a cutting force estimate that properly incorporates the geometry and timing of the tool-workpiece interaction. The cutting force is modeled as the product of a material-dependent cutting coefficient and the instantaneous area of the uncut chip. In precision flycutting the tool radius $R$ is large compared to the feed $f$ and depth of cut $h$, so the instantaneous area of the uncut chip is approximately $f h$. The depth of cut $h$ may be considered as the sum of the desired depth of cut $h_d$ plus the time-varying deflection in the structural loop $x$. The contribution of structural vibration $x$ in the cutting force provides feedback to the system. Figure 1 shows a sample workpiece and the key geometrical parameters needed to predict the cutting force.

![Figure 1 Schematic of the diamond tool, workpiece and tool paths.](image)

The cutting force $f_i$ in the direction normal to the workpiece is

$$f_i = -K_f f h = -K_f (f h_d + x)$$

The cutting coefficient $K$ is on the order of one kN/mm$^2$ for nonferrous metals such as aluminum. Lucca and Rhorer have published data for the cutting coefficients of several likely diamond machinable materials at the low depths of cut that are typically found in precision flycutting (Lucca 1991, Lucca 1994).
A linear, time-invariant, lumped parameter model of the cutting tool and holder, the spindle, the machine axes and base, and the workpiece with fixturing captures the relevant dynamics. In many cases, a single degree of freedom model will successfully represent the key behavior while simplifying the analysis to provide insight into the important results. We provide both a general analysis and a specific solution using a single degree of freedom representation of the structural loop.

The transfer function relating the tool-workpiece deflection to the cutting force in the Laplace domain is

\[ \frac{X}{F} = G(s) \]  

(2)

The vibration-dependent cutting force results in a feedback term in the transfer function between the desired \(H_d\) and actual \(X\) depth of cut.

\[ \frac{X}{H_d} = -\frac{GKf}{1 + GKf} \]  

(3)

Of course, this model is valid only when the tool is in contact with the workpiece. When not in contact with the workpiece, the cutting force is zero and the response is determined by the state of the system at the moment the tool leaves contact with the workpiece.

For the special case of a single degree of freedom model, which is valid for hardware configurations with a dominant mode of vibration, the system dynamics may be written as

\[ m \ddot{x} + c \dot{x} + (k + \delta K f)x = -K f h_d \delta \]  

(4)

where \( m \), \( c \), and \( k \) are the modal mass, damping, and stiffness associated with the dominant structural resonance. The variable \( \delta \) is used to toggle the intermittent contact of the workpiece and tool and is defined as unity during contact and zero the remainder of the time.

Equation 3 or 4 may be solved numerically to predict the tool vibration during flycutting. Figure 2 shows a sample solution over two revolutions of the flycutter. In this particular example, cutting occurs during 80 degrees of the tool rotation. During the contact between the tool and workpiece, the cutting force remains essentially constant within the resolution of the plot because the stiffness of the machine is large compared to the feedback term \( K f \). For many practical flycutting situations the contribution of this feedback term is small compared to the structural stiffness.
Neglecting the feedback term allows a significant simplification: the entire problem may be solved by Fourier series analysis. In this case, the solution times become very short and many different scenarios can be quickly compared. The periodic forcing function captured by the variable \( \delta \) that alternates between zero and unity may be written as:

\[
\begin{align*}
\sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t
\end{align*}
\]

where \( \alpha \) is the fraction of the total time per revolution in which the tool is cutting. The function \( \delta \) may be rewritten with a magnitude \( A_n \) and phase angle \( \beta_n \).

\[
\begin{align*}
A_n &= \sqrt{a_n^2 + b_n^2} \\
\beta_n &= \tan^{-1}\left(-\frac{b_n}{a_n}\right)
\end{align*}
\]

The single degree of freedom dynamics of Eq. 4 can be readily solved using the forcing function of Eq. 6.
\[
X_n = \frac{A_n}{\sqrt{(k - m(n\omega)^2) + (n\alpha)^2}} \\
\phi_n = \tan^{-1}\left(\frac{-n\alpha}{k - m(n\omega)^2}\right) \\
X(t) = \frac{Kf_n}{k_c}\left\{\alpha + \sum_{n=1}^{\infty} X_n \cos(n\omega t + \beta_n + \phi_n)\right\}
\]

The most meaningful figure of merit for flycutting and interferometric evaluation of workpieces is the surface figure obtained by a particular set of machining parameters. The RMS surface figure is found by integrating the square of the tool vibration over the arc in which the workpiece and tool are in contact.

\[
\text{RMS surface figure} = \frac{1}{T} \int_{0}^{T} x^2(t) \, dt
\]  

Eq. 8 cannot be solved in closed form because the sinusoidal components of the tool vibration are not being integrated over an integer number of oscillations. However, a numerical solution is readily obtained with sufficient accuracy using on the order of \(10^2\) to \(10^3\) terms of the Fourier series (series convergence goes with \(n^{-1}\) as shown in Eq. 5). Specific numerical examples will be presented in the sections that follow and compared to actual interferometer results.

A more general solution to single or multiple degree of freedom models may be obtained by numerical integration of Eq. 3. The key issues are identical to the SDOF solution outlined above. In the case of a numerical solution to the general model it is important to synchronize the time steps to include the precise moment that the tool first engages and leaves the workpiece in each rotation so that a converged solution can be found with the fewest possible integration steps.

**Experimental Setup**

Figure 3 shows the two-axis machine tool with a 15 kRPM air bearing spindle used to prepare the workpieces to experimentally validate the cutting and surface finish model (Professional Instruments model 9590 flycutting machine and Dover track writing spindle). The depth of cut used in all testing is two micrometers and the feed per spindle revolution was kept constant for all spindle speeds at 50 micrometers. With the 3 mm radius diamond tool used in this testing (single crystal, zero rake tool from Edge Technologies), the theoretical surface finish, in the absence of structural vibration, is 100 nm peak-to-valley and 30 nm \(R_m\).
Figure 3 Machine hardware used to validate the flycutting model and prepare coupons for interferometric inspection.

Figure 1 showed a close-up view of a 6061-T aluminum workpiece and diamond tool that is mounted on the fixturing shown in Figure 3. As can be seen in the figure, the arc-shaped path of the rotating tool matches the entrance and exit radii of the workpiece so that the loading on the tool is constant for the entire test. This simplifies the interpretation of the interferometer results by ensuring that the cutting force-induced out-of-flatness is constant over the entire workpiece. Figure 1 also shows a cross-sectional view of the workpiece, including the shaded area of the material removed in the current pass of the tool over the workpiece. This area, when multiplied by a material-dependent cutting coefficient, gives a good prediction of the cutting force on the tool, as described in Eq. 1.

The dynamics of this specific machine are dominated by a structural resonance at 300 Hz. The relevant modal parameters were determined from the experimentally measured frequency response function shown in Figure 4. The dominant mode has a damping factor of 1% as calculated by polynomial curve fit. The static stiffness is 20 MN/m.
Workpiece Evaluation

The flycut samples were measured for flatness using a phase shifting interferometric technique with a transmission flat set up to form a Fizeau measurement cavity with the surface of the test coupon (Zygo VeriFire AT interferometer). The illumination wavelength was 632.8 nm and 1k x 1k spatial sampling resolution was used. The VeriFire contains Zygo’s Ring of Fire™ illumination system that suppresses coherent artifacts (interference “bullseyes”) formed by dust or other defects outside of the test cavity, therefore enhancing measurement performance.

Two similar set-ups were used to make the measurements. Because of the high-reflectivity diamond turned aluminum surface, a 4 inch flat with a 20% reflectivity Dynaflect coating was used in the first set-up to improve fringe contrast in the measurement. In the second set-up, a standard 4% reflectivity transmission flat was used and an attenuation filter was inserted into the measurement cavity to reduce the intensity of the reflected beam from the sample to again improve fringe contrast. In both set-ups, the length of the measurement cavity was minimized to reduce environment effects in the measurement. Each sample was manually aligned using an accessory mount in 2 axes of tilt to the test cavity. The surface error of both transmission flats was specified as λ/20. Only tip, tilt, and piston were removed from the measured data. No additional filtering of data or measurement averaging was used at the time of measurement. All measurement data were exported to ASCII files for post processing. In MATLAB, a 4th order polynomial was removed from the surface to eliminate the residual effects of fixturing deformation on the measured figure error. A low pass filter was also applied in the feed direction to attenuate the influence of the flycutting machine’s ball screw.

Figures 5 and 6 show the measured and predicted workpiece waviness at flycutting spindle speeds of 690 and 820 RPM. The dominant structural frequency is independent of the intermittent cutting forces so the
flycutting spindle speed directly influences the number of waves generated on the workpiece as predicted by the model.

Figures 5 and 6 provide a side-by-side comparison of the actual surface of the flycut workpiece, as measured by phase shifting interferometry, to the results predicted by the analytical model presented above. The analytical results are obtained by numerically integrating Eq. 3 using parameters matching the actual cutting conditions. Comparison of the measured and predicted results is conveniently done by visual inspection because the predicted tool vibration can be mapped to the shape of the workpieces.

**Discussion**

The interferometrically-verified model may be used to efficiently explore a variety of issues that may be encountered in practical precision flycutting applications. In this section, the role of structural loop stiffness,
damping, and material properties will first be discussed. The relationship between workpiece size, spindle speed, and resonant frequency is then studied so that practitioners may identify the machining parameters that will result in workpieces with reduced out-of-flatness. It will be shown that a simple change in spindle speed can have a large effect on the workpiece flatness.

If the dynamics of the flycutting machine’s structural loop are reasonably represented with a single dominant mode of vibration, the model of Eq. 4 offers very fast solution time and generates significant insight into the dynamics of precision flycutting. Furthermore the model may be partially non-dimensionalized so that the dominant frequency of vibration in the machine is normalized by the spindle speed to obtain a more general result. Figure 7 shows the predicted workpiece flatness obtained by running the model over a wide range of normalized spindle speeds. The peak (i.e., worst case) flatness values correspond to spindle speeds that are integer sub-multiples of the resonant frequency. At these speeds, the timing of the interrupted cut is such that waves build up on the surface of the workpiece with the highest amplitude. It is important to note that even at the spindle speeds with the worst flatness the cutting system is stable, and not experiencing regenerative chatter; rather the results show an unfortunate combination of machining parameters that result in particularly poor quality workpieces.

![Figure 7 Effect of structural loop stiffness on workpiece flatness.](image)

Between each peak in the RMS flatness values is a local minimum that represents a much more favorable cutting speed for a given set of machine dynamics and workpiece geometry. These local minima are the preferred speeds of operation regardless of damping, machine stiffness, and workpiece material.
Figure 7 also shows the linear relationship between the structural loop stiffness of the flycutting machine with the RMS flatness. Any increase in machine stiffness leads to a proportional decrease in the workpiece surface figure error (i.e., out-of-flatness) provided that the spindle speed maintains a constant ratio with the resonant frequency.

Figure 8 shows the effect of flycutting at varying material removal rates (realized by changing the feed per revolution \( f \) or the desired depth of cut \( h_d \)). A similar trend is also seen with variations in the cutting coefficient of the workpiece material \( K \). Changes in either the material removal rate or the cutting coefficient \( K \) cause a proportional change in cutting force, as shown in Eq. 1. The result of either of these changes is a linearly proportional variation in the workpiece flatness.

Figure 8 Effect of depth of cut or material cutting stiffness on workpiece flatness.

Figure 9 shows the effect of structural damping on the workpiece flatness. As is the case with other forms of forced vibration, damping reduces the vibration when the excitation frequency is at or near the resonant frequency. The damping is somewhat undesirable if the process could be reliably run at the precise spindle speed of an anti-resonance of Figure 9 (e.g., spindle speed of 78% the dominant resonant frequency is one such speed). In practice, it is probably more beneficial to obtain damping wherever possible to increase overall robustness.
The final result to come out of the flycutting model is a comprehensive picture of the relationship between the spindle speed, the dominant resonant frequency, and the arc of contact with the workpiece. Figure 10 shows the RMS flatness obtained by workpieces of various sizes when cut at various speeds. The results indicate that, in general, it is highly undesirable to set up a flycutting process such that the dominant resonant frequency is an integer multiple of the spindle speed (the opposite is not true; the spindle speed may in fact be chosen at a multiple of the natural frequency although speeds this high are unlikely to be realizable in practice). When the natural frequency is chosen as a multiple of the spindle speed, the vibrating cutting tool will make an integer number of oscillations per revolution. Depending on the size (i.e., arc) of the workpiece, this can lead, in some situations, to an intermittent cutting force timed such that the vibration amplitude becomes large.

If we consider the RMS flatness results shown in Figure 10 for the specific spindle speed that is one-third that of the resonant frequency, we find two maxima at 60 and 180 degrees of contact arc. For workpieces of either of these two sizes the tool is vibrating such that it leaves the workpiece out of phase and therefore with the greatest net excitation, as shown in Figure 11. The arrows in Figure 11 indicate the portion of the total time that the tool is in contact with the workpiece. In these cases, the relationship between the spindle speed and resonant frequency is such that \( n + \frac{1}{2} \) waves are generated on the workpiece where \( n = 0, 1, 2, \ldots \).

Between these two maxima is a minimum in the workpiece out of flatness when the arc of contact is 120 degrees. For workpieces of this size, the tool is vibrating such that the tool leaves the workpiece in phase with its entrance and with the least net energy imparted to the tool. In this best-case situation, \( n \) waves are generated in the workpiece.
Figure 10 Effect of workpiece size on flatness.
Conclusion

This paper has shown how interferometer measurements may be used to generate a reliable method of optimizing an ultra-precision flycutting process. In the course of this work, a model was made and verified experimentally to explore how the structural loop dynamics and periodic loading interact to effect workpiece flatness. This model is a useful tool to improve the resulting flatness of a flycutting operation based on the workpiece geometry and spindle speed. The results provide insight into important considerations for the selection of machining parameters and for the selection of the most appropriate spindle speeds for a given workpiece.

The results show that the preferred spindle speed in ultra-precision flycutting should be chosen such that the dominant structural resonance does not occur at an integer multiple of the spindle speed. Furthermore, the spindle speed can be fine-tuned such that the vibrating tool will make an integer number of oscillations when the tool is in contact with the workpiece. The exact phasing of the process conditions can be verified by interferometer inspection of trial workpieces.
Acknowledgement

The authors wish to thank Dr. Carl Zanoni and the Zygo Corporation for the use of the VeriFire AT interferometer on which measurement for this work were made.

References


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