A METROLOGY ANALYSIS OF THE SPHERICITY
OF THE PROSTHETIC FEMORAL HEAD

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Abstract

In the medical field today, total joint replacement surgeries are fairly routine. A substantial portion of these procedures is in fact a result of the limited life of the joint prosthesis, making improvements in the lifespan of these devices a necessity. To make progress in this area, it is essential to first determine the actual shape of the femoral head. Though much research has been conducted, there are many inconsistencies concerning the true surface characteristics and actual contour of the femur. As such, the objective of this research is to use a surface metrology approach to evaluate the sphericity or otherwise of the prosthetic femoral head.

Using precision spindle metrology, femoral head surface measurements were taken at seven test positions within the upper hemisphere of the joint. These measurements show good repeatability, even when the spindle speed was varied. Using the Fourier Series, the spindle error motion and other mounting errors were removed from the data, leaving the artifact error for analysis. These testing results indicate that although the error ranges about 0.5 thousandths of an inch, the average error of all data is 0.1314 thousandths. Because this error is extremely small, it is accurate to say that the prosthetic femoral head is seemingly round. However, magnification of the surface error also proves there is a distinct two-lobe shape to the hip joint.
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Chapter 1: Introduction

This chapter introduces the basic concepts and terminology of the femoral head which will be used throughout this thesis. The objective of this work will be discussed, detailing the motivation behind the research. Additionally, this chapter will include a brief overview of the previous work done in determining the shape of the femoral head. Different measurement techniques for determining surface topography will also be explained and recommendations made regarding best methods for accurate measurement.

1.1 Introduction

In the medical arena today, hip replacement surgery is fairly common. An estimated 200,000 orthopedic joint prostheses are implanted annually in the United States [1]. Though the surgical techniques and treatments are highly sophisticated, wear of the joint is inevitable. In fact, the current expected lifespan for prosthetic joints does not exceed 15 years [2]. There are also huge economic implications as annual costs of hip and knee revision surgeries exceed $2.4 billion (and are projected to top $7 billion in the next 20 years). With over 60,000 revision surgeries performed each year and due largely to wear, there is a definite need for prostheses with increased lifespan [2].

The wear mechanism of the hip joint involves the perpetual rubbing of the femoral head against the acetabular cup during movement. The upper end of the femur, more commonly referred to as the femoral head, fits with the cup-shaped socket of the acetabulum as illustrated in Figure 1.1 [3]. The head of the femur articulates with the acetabulum forming a ball-and-socket type joint. This relative motion has been identified as a major factor affecting the wear of the prostheses. Specifically, researchers have established
that the characteristics of the surface topography of the femoral head are the primary determinant of wear rate [1].

**Figure 1.1: Anatomy of human hip joint, depicting ball and socket fit of the femoral head and the acetabulum [3]**

To improve the lifespan of the prosthetic hip joint, it is thus critical to first examine and classify the shape of the femoral head. There has been much debate regarding the true surface characteristics and actual contour of the femur. As such, the objective of this research is to use a surface metrology approach to evaluate the sphericity or otherwise of the prosthetic femoral head, in order to gain a more solid understanding in improving the lifespan of the joint.

### 1.2 Previous Work

Over the past century, researchers have attempted to identify and establish a standard model for the human femoral head. Early research (Aeby, 1863; Schmid, 1876; Helwig, 1912;
Walmsey, 1928) employed comparative methods and found the head of the femur distinctly aspherical [4]. In his testing, Walmsey concluded that the head is in fact more of a rotational ellipsoid, which varies in size from individual to individual [5].

However, these methods were reconsidered in the late 1960s with the work of Hammond and Charnley. In retesting these old techniques in combination with other methods, they found the femoral head has only inconsistent patterns of extremely small deviations from a true sphere. Their work asserted the hip joint to be remarkably spherical, discounting all previously published data [5].

Most recent published work now suggests the hip joint to be a rotational conchoid shape whose radius is described by the general formula [6]:

\[ r = a + b \cos \varphi \]  

(1.1)

Figure 1.2 depicts the conchoid shape. Menschik tested this hypothesis using a high precision CNC-coordinate measuring machine to measure the mean deviation from both a sphere and a conchoid. Though he found both shapes provide a reasonable fit for the femoral head, his work established that this deviation showed a better fit for a conchoid shape compared to a sphere [6].

![Conchoid Shape Diagram](image)

**Figure 1.2:** Conchoid shape with the general equation \( r = a + b \cos \varphi \) [6]
There is also much discussion surrounding best methods and practices to achieve precise measurement of the hip joint. Clarke et al. tested the validity of using three-legged micrometer gauges in measuring the human hip joint [4]. This study concluded that, though commonly used in industry for “detecting variations in surface curvature,” spherometer devices are inherently unreliable in taking surface measurements of the femoral head.

In a similar manner, Blunt and Jiang conducted a more comprehensive study of best methods for the three-dimensional surface metrology of orthopedic joint prostheses. Their work examined the use of contacting stylus instruments, atomic force microscopes, phase-shifting interferometers, and other non-contacting devices. Blunt and Jiang’s work revealed any limitations and specified appropriate use of each method. They concluded that stylus instruments should only be used when possible to correct for table error. Additionally, this research showed that atomic force microscopy should be used when a more detailed surface description is necessary. Their work suggests that the high resolution and large range of phase-shifting interferometers offer the best means of surface measurement [1].

Though several methods of surface metrology have been analyzed extensively and in relation to joint prostheses, it is still unclear as to the exact shape of the femoral head. In short, some researchers agree the femoral head is spherical while others report it is distinctly aspherical. This lack of conclusive evidence provides motivation for this research, with hopes of eventually defining how surface characteristics affect the lifespan of hip joint prostheses.
Chapter 2: Error Terms Analysis

This chapter discusses the error terms analysis used to identify and describe the out-of-roundness error of the prosthetic femoral head. The fundamentals of spindle metrology and the classification of spindle error motion will be explained. This chapter will largely focus on the methodology used to characterize and quantify error, including the application of the homogenous transformation matrix (HTM). In addition, the results of this HTM analysis will be explained and the error motion summarized.

2.1 Spindle Metrology and Error Motion

In the field of precision spindle metrology, it is essential to determine the difference between the surface of an artifact and the spindle error motion itself. Ideally, a spindle can only rotate about a single axis. Thus motion in all other degrees of freedom is considered unwanted and can be classified as either spindle error or error due to external sources [7]. As such, several error separation techniques have been developed to categorize this error motion and isolate the data of interest.

Though spindle error can be measured in many ways, it is often defined in terms of three specific categories – spatial measurement, frequency, and sensitive direction. Spatially, error motion is classified into components of axial, radial, tilt, and face terms. These elements determine the rotational accuracy of the spindle and depend upon the orientation of the sensor. In terms of frequency classification, error motion of the spindle can be separated into two categories: synchronous and asynchronous error. As Figure 2.1 shows, synchronous error motion is described by the frequency components occurring at integer multiples of the fundamental frequency (which describes the speed at which the spindle
rotates). Non-integer multiples of the fundamental are classified as asynchronous [7]. In an FFT analysis, it is often convenient to normalize the frequency content by this fundamental, producing a dimensionless quantity expressed in units of undulations per revolution (upr).

By definition, the fundamental frequency is 1 upr.

**Figure 2.1:** Synchronous and asynchronous components of error motion [7]

A third and final approach is the classification of spindle error in terms of the sensitive direction. Motion in the sensitive direction is defined as movement in the “line of action” of the sensor or gage. This orientation can either be fixed or rotating. For the first case, the part is typically rotated relative to a stationary gage, meaning the sensitive direction has a fixed orientation with respect to the spindle. In contrast, a rotating sensitive direction implies that the gage is rotated relative to a fixed part. In this case, the error motion of interest is the rotating sensitive direction [8].
When the out-of-roundness error of the artifact cannot be ignored (as is the case with this femoral head study), it is necessary to further analyze the synchronous error motion. There are several techniques used to separate error including reversals, multistep methods, and multiprobe methods. Each method will yield like results providing the recorded error measurements are repeatable. Thus an ideal separation “correctly assigns all synchronous components in the recorded data to the spindle or artifact in the correct proportion” [7]. These methods will be employed in the subsequent error terms analysis.

2.2 HTM Error Analysis

To quantify the geometric error present in the test setup, the relative positions of the workpiece and the probe were compared. Any difference in these global positions would therefore describe the systematic error to be removed from the test data. The difficulty in defining position with respect to two different orientations, or coordinate frames, is easily reconciled with the use of the homogeneous transformation matrix. This section describes the functions of the matrix and how it was applied to determine the geometric error in the femoral head measurement.

2.2.1 The Homogeneous Transformation Matrix

The basis of this analysis utilizes the homogenous transformation matrix (HTM) to determine the spatial relationship between the femoral head and the probe measuring its sphericity. Simply put, this 4x4 transform matrix is used to express the position of a rigid body in space with respect to a given coordinate system [9]. The HTM, denoted $^R T_s$, transforms the three-dimensional coordinates of the rigid body frame $(X_s, Y_s, Z_s)$ to the
desired reference coordinate system \((X_R, Y_R, Z_R)\). Thus the coordinate transformation of a point in the coordinate frame \(n\) with respect to a reference frame \(R\) can be expressed as:

\[
\begin{bmatrix}
X_R \\
Y_R \\
Z_R \\
1
\end{bmatrix} = T_n \begin{bmatrix}
X_n \\
Y_n \\
Z_n \\
1
\end{bmatrix}
\]

(2.1)

The pre-script of the HTM represents the reference frame desired (in this case \(R\)), and the post-script represents the original coordinate frame from which you are transforming (in this case \(n\)). Equation 2.2 represents the general form of the HTM. The first three columns specify the orientation, \(O\), of the rigid body’s axes with respect to the reference frame. These entries are known as the directional cosines and are expressed in terms of unit vectors \(i, j, k\). The final row of these columns indicates a scale factor of 0. The last column represents the position of the origin of the rigid body’s coordinate system with respect to the reference frame \((P_x, P_y, P_z)\). The final entry \(P_s\) is a scaling factor and is usually set to unity [9].

\[
R T_n = \begin{bmatrix}
O_{ix} & O_{iy} & O_{iz} & P_x \\
O_{jx} & O_{jy} & O_{jz} & P_y \\
O_{kx} & O_{ky} & O_{kz} & P_z \\
0 & 0 & 0 & P_s
\end{bmatrix}
\]

(2.2)

The HTM can also be used to represent a series of coordinate transformations. For the simultaneous combination of these axes transforms, the corresponding HTMs can be multiplied in series to obtain a single transformation matrix. These intermediate HTMs describe the relative position of each axis to each intermediate coordinate frame in the series [9]. This utility is especially useful for structures that can be easily decomposed into a series of coordinate transformations, and as such, is the basis for the following error terms analysis.
2.2.2 Evaluating Error Using HTM

In a simplified test setup, illustrated in Figure 2.2, the hip joint is mounted to a precision spindle that rotates about its center an angle $\theta$, varying between $0$ and $2\pi$. The sphericity of the femoral head is measured using a contacting ruby tipped stylus that is oriented at a fixed angle of $45^\circ$. As the femoral head rotates, a 1 mm/V capacitance probe reads and records the displacement of the stylus as a voltage. The workpiece is traversed in the vertical Z-direction and the probe passes in the horizontal X-direction. The details of this setup will be further explained in Chapter 3.

![Figure 2.2: Test setup illustrating axes of rotation](image)

To simplify the following analysis of the sphere side axes transformation, the reference frame abbreviations of Table 2.1 will be used.
Sphere Side Coordinate Frames

| Ref = center of vertical spindle stator |
| S1 = spindle rotor (coincident with Ref) - rotation angle \( \theta \) |
| S2 = spindle rotor (coincident with Ref) - assume error motion = 0 |
| S3 = workpiece height (to center of sphere) - \( H \) |
| S4 = workpiece eccentricity (mounting offset) - in X (sx) and Y (sy) directions |

### Table 2.1: Reference frame abbreviations for the sphere side analysis

The first axes transformation accounts for the spindle rotation angle \( \theta \). The HTM of Equation 2.3 thus transforms the coordinates of a point by an angle \( \theta \) about the Z-axis.

\[
^s_{T_{\text{Ref}}}^{\theta} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 & 0 \\
\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(2.3)

Similarly, the other three HTMs of this series transformation are defined. Because the spindle error motion is assumed to be zero, the HTM transforming the coordinates from reference frame S1 to S2 is simply an identity matrix. To account for the height of the workpiece itself, the HTM of Equation 2.4 is used. A final HTM is applied to incorporate the X and Y-direction mounting offsets (denoted \( sx \) and \( sy \)). This workpiece eccentricity transform is expressed in Equation 2.5.

\[
^s_{T_{s2}}^{\theta} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & H \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(2.4)

\[
^s_{T_{s3}}^{sx} = \begin{bmatrix}
1 & 0 & 0 & sx \\
0 & 1 & 0 & sy \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(2.5)
Applying the principle of simultaneous motion, these HTMs are multiplied in series to form a single transformation matrix. This HTM is now a transformation from the original coordinate frame, Ref, to the final coordinate frame, S4. Thus a point corresponding to the origin of the Ref frame (0, 0, 0) can easily be transformed to the S4 frame through the matrix multiplication of this point (and a scaling factor of 1 as a fourth entry) with the HTM. The product is a 4x1 vector that determines the position (X, Y, Z, 1) of this coordinate in the S4 frame. Equation 2.6 expresses this position in vector form.

\[
\begin{bmatrix}
X_{S4} \\
Y_{S4} \\
Z_{S4} \\
1
\end{bmatrix} =
\begin{bmatrix}
sx \cdot \cos \theta - sy \cdot \sin \theta \\
sy \cdot \cos \theta + sx \cdot \sin \theta \\
H \\
1
\end{bmatrix}
\]

(2.6)

A similar analysis was performed to determine the probe side position in the desired reference frame. Table 2.2 represents the coordinate frame abbreviations used.

<table>
<thead>
<tr>
<th>Table 2.2: Reference frame abbreviations for the probe side analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 = center of probe body w.r.t. reference point (includes mounting offsets)</td>
</tr>
<tr>
<td>P2 = probe body rotation (coincident with P1) - (-135°)</td>
</tr>
<tr>
<td>P3 = probe body mounting errors (coincident with P1) - pax, pay, paz</td>
</tr>
<tr>
<td>P4 = probe length (d)</td>
</tr>
</tbody>
</table>

Beginning with the same reference frame (Ref, as defined in the sphere side analysis) the HTM of Equation 2.7 transforms coordinates from the spindle center to the probe center. This transform includes both X and Z-direction linear translation and mounting errors (offx, offy, offz). The next HTM of the series takes into consideration the orientation of the probe, which is fixed at an angle of -135° (or -45° from the horizontal). Equation 2.8 expresses this HTM which transforms a point from the center of the probe body, or P1 frame, to the rotational frame P2. The HTM corresponding to the transformation from the
P2 frame to P3 frame accounts for mounting errors of the probe (denoted \( pax, pay, paz \)) and is expressed in Equation 2.9. Finally, the probe length \( d \) is taken into account; Equation 2.10 represents the HTM for this linear transformation.

\[
\begin{align*}
\begin{bmatrix}
1 & 0 & 0 & \text{offx} + X \\
0 & 1 & 0 & \text{offy} \\
0 & 0 & 1 & \text{offz} + Z \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]  \hspace{1cm} (2.7)

\[
\begin{align*}
\begin{bmatrix}
-1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\
0 & 1 & 0 & 0 \\
1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]  \hspace{1cm} (2.8)

\[
\begin{align*}
\begin{bmatrix}
1 & -paz & pay & 0 \\
paz & 1 & -pax & 0 \\
-pay & pax & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]  \hspace{1cm} (2.9)

\[
\begin{align*}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & d \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]  \hspace{1cm} (2.10)

Again, these four transformation matrices are multiplied together to produce a single HTM that transforms any point in the coordinate frame Ref to the desired P4 coordinate frame. The same point \((0, 0, 0, 1)\) of the original Ref frame is multiplied by this probe side HTM to determine the equivalent P4 frame coordinates. This transformation yields the position \((X, Y, Z, 1)\) in the P4 frame, as shown in Equation 2.11.
Using the results of this HTM analysis, the distance between the two points in the S4 coordinate frame (Equation 2.6) and the P4 coordinate frame (Equation 2.11) was found. The numerical difference in these points should be equivalent to the sum of the radii of the sphere and the probe plus the synchronous error:

$$\text{distance}_{S4-P4} = r + R + \text{error}$$

Equation 2.11 expresses this difference in terms of $r$, the radius of the probe, and $R$, the radius of the femoral head.

## 2.3 Results

This HTM analysis has allowed for the estimate of various geometrical error sources. To recap, $\theta$ is the spindle rotation angle, $d$ is the probe length, and the variable $\phi$ is the latitude of the probe, varying from $0$ at the top of the sphere to $\pi/2$ at the side. Table 2.3 summarizes these findings, listing the coefficients corresponding to the different forms of error. The sine and cosine terms will vary in magnitude up to a value of 1. Thus the magnitude of these geometrical errors will map to measurement error on a one-to-one basis.
<table>
<thead>
<tr>
<th>Offset in probe location (X-dir)</th>
<th>( \cos \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offset in probe location (Y-dir)</td>
<td>0</td>
</tr>
<tr>
<td>Offset in probe location (Z-dir)</td>
<td>( \sin \phi )</td>
</tr>
<tr>
<td>Probe mounting angle error (from 0 about X-axis)</td>
<td>0</td>
</tr>
<tr>
<td>Probe mounting angle error (from 45° about Y-axis)</td>
<td>( d (-\cos \phi + \sin \phi)/\sqrt{2} )</td>
</tr>
<tr>
<td>Probe mounting angle error (from 0 about Z-axis)</td>
<td>0</td>
</tr>
<tr>
<td>Artifact eccentricity (X-dir)</td>
<td>( -\cos \phi \cos \theta )</td>
</tr>
<tr>
<td>Artifact eccentricity (Y-dir)</td>
<td>( -\cos \phi \sin \theta )</td>
</tr>
<tr>
<td>Probe tip radius</td>
<td>1</td>
</tr>
<tr>
<td>Femoral head radius</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 2.3:** Coefficients of various geometrical error terms

Once these offsets and mounting errors are accounted for, the femoral head sphericity can be directly measured. The test procedure used to determine the actual shape of the femoral head will be explained in Chapter 3.
Chapter 3: Methods and Measurement of the Femoral Head Sphericity

This chapter presents an in-depth explanation of the methods used to assess the femoral head shape. A brief overview of the instrumentation and hardware will be provided along with a complete description of the experimental setup. The steps taken in determining the femoral head radius will be identified. The chapter will go on to explain how this data were manipulated to obtain the desired positions for data collection. The final sphericity measurements will collectively be analyzed and a conclusion addressing the actual shape of the hip joint will be drawn.

3.1 Instrumentation, Hardware, and Experimental Set-up

The hip joint under investigation, pictured in Figure 3.1, is the titanium McKee Farrar prosthesis manufactured by Howmedica Osteonics Corporation (now a division of Stryker Orthopaedics).

![Figure 3.1: The McKee Farrar prosthesis (photograph and CAD model)](image)
To accurately measure the surface of the femoral head, the prosthesis was mounted to a cylindrical chuck on an air-bearing spindle. Using a vertical mill, a rectangular slice was removed from chuck, as to protect the integrity of the hip joint. This small gap in the chuck allows enough room for the contour of the femur, so that the joint can be loosely placed in the chuck with the head of the hip joint resting upon the top surface. The milled chuck and hip joint/chuck assembly is illustrated in the CAD models of Figure 3.2. The chuck was then mounted to the precision spindle. The idea in this setup was to have the spherical head of the joint centered on the chuck and spindle.

![Figure 3.2: The spindle-mounted chuck, shown above with the rectangular cut and with the loosely fixed hip joint](image)

This chuck was mounted to the water-cooled Professional Instruments Twin-Mounted spindle of the Moore AG150 Aspheric Generator, pictured in Figure 3.3. A dial indicator was used to tap-in the workpiece with an eccentricity of approximately 0.0005 inches. At this point, the chuck was further centered with a high precision capacitance probe. Capacitive sensors are commonly used in precision metrology as they provide fine resolution over a wide range of frequencies [7]. The chuck was then properly fastened to the spindle.
The next challenge was trying to accurately position the hip joint on the centered chuck. Clay was used to temporarily fix the joint to the chuck. This material kept the workpiece in place while allowing for some slight movement in centering. Again, the sphere was tapped in using the same dial-gauge and capacitance probe instrumentation. Once approximately centered, the femoral head was permanently attached to the chuck with a small amount of epoxy, distributed evenly along the edge of the femoral head base. Though it is impossible to perfectly center the workpiece on the spindle, these methods have ensured very small radial errors motion (which can easily be removed as discussed in Chapter 2).

![Figure 3.3: The Moore AG150 Aspheric Generator with water-cooled Professional Instruments Twin-Mounted Spindles](image)

It is important to note that with the Moore AG150, the X and Z-motions are independently controlled. That is, the table can move linearly in the horizontal X-direction only, while motion along the vertical Z-axis is determined by the spindle location. There is no movement in the normal Y-direction. The Moore AG150 also directly controls the rotational speed of the spindle $\omega$ and the rotation angle $\theta$ itself.
To measure the surface characteristics of the centered femoral head, a non-contacting capacitive sensor was used in conjunction with a contacting ruby-tipped stylus, 0.125 inches in diameter. A capacitance probe is basically half of a parallel plate capacitor with air acting as the dielectric medium. The Air Bearing C-LVDT, pictured in Figure 3.4, supports the movement of a conductive plunger that acts as the target surface for the probe to read. Basically this device is used to convert a contacting stylus sensor into a non-contacting capacitive sensor [7]. In this test setup, a Lion Precision capacitance gauge and driver with a sensitivity of 1 V/thousandth of an inch was used. This Air Bearing device was held in place by a bracket angled 45° south from the horizontal. The bracket was fixed to an arm mounted on the horizontal axis of the Moore AG150 and positioned in the Y-direction at an approximate center location.

![Figure 3.4: The Lion Precision Air Bearing C-LVDT converts a capacitive sensor into a stylus contacting sensor [7]](image)

In terms of data acquisition and hardware, the signal produced from 1 V/mil capacitance probe was captured using the Virtual Capture (VCAP) feature of the Siglab
software. This program allows the collected data to be recorded and imported easily into Matlab for further analysis. Additionally, a spindle-mounted encoder was used to trigger the sampling. This rotary encoder provides a clear pulse in the data once-per-revolution. The test setup in its entirety is pictured in Figure 3.5.

![Figure 3.5: The complete experimental setup](image)

### 3.2 Procedure

This section of the chapter describes the procedure used for collecting data. Before any measurements were taken however, it was important to establish a “homed” position in the X and Z-directions. This “homed” position of the testing setup was marked with tape and tested to ensure reproducibility. After this, the setup was tested in a trial run. Raw data were collected for three different latitudes of the sphere, recording both the probe voltage and the encoder signal over time. The data collected from the first of these test locations are plotted in Figure 3.6.
Figure 3.6: Pretest data depicting repeatability of shape over 3 revolutions

This plot clearly shows a repeatable shape for each full revolution. A sine waveform is also apparent (and expected) in this plot due to the small offsets of the imperfect centering of the femoral head on the spindle. The same data were taken at different latitudes of the sphere, and these measurements proved similar results. These initial findings were necessary to ensure the validity of the experimental setup. They also indicate that the femoral head has a definite shape which may include some imperfections in the seemingly round form.

3.2.1 Femoral Head Radius Measurement

The first task in obtaining repeatable surface data was to define the geometry of the workpiece in relation to the homed X and Z-positions of the machine. Basically, this meant determining the precise radius of the femoral head. This was accomplished by finding the
positions that produce maximum displacement at the top and side of the sphere. Using simple geometry, the radius of the femoral head could be extracted from these local maxima. This radius measurement is important in determining appropriate locations for collecting surface data. Since the Moore AG150 allows for programmable motion, global testing locations in the X-Z plane could be specified.

To determine the top-most point, the probe was swept across the femoral head a distance of 0.020 inches in the X-direction while held at a set Z-position. This small range was found by trial and error, using a capacitance probe and the instantaneous output from the VCAP oscilloscope. The distance of the sweep was narrowed until the maximum was known to exist within a 0.020 inch range. The displacement data from the probe was then recorded and the maximum value extracted from the data. Similarly, the exact side position of the sphere was found sweeping the probe 0.020 inches in the vertical Z-direction, keeping the X-position fixed.

Though the coordinate positions of the machine are known at all times, there is no simple way to transmit this information to the data acquisition system. Thus presents the challenge of recording both data and position simultaneously. In a first attempt to compute the top and side positions of the sphere, two capacitance probes were used: one to measure the surface of the sphere, and one to measure the linear (X or Z) position of the machine. However, this method proved inefficient and the results were inherently imprecise. This data was discarded and a new method was formulated to more accurately record the machine’s position.

The global positions of these maxima were ultimately established by recording the data in steps. A G-code was written to move the probe in steps across the same 0.020 inch sweep described above. These steps allowed the probe to move to a specified location, stop
completely, and dwell at that location for a small amount of time. This is shown in the
displacement vs. time plot of Figure 3.7. This dwell time ensures the probe reaches the exact
location, making it possible to track the position of the machine. In this procedure, twenty
steps were taken in increments of 0.001 inches for both the top and side measurements.

Figure 3.7: Plot of step data taken at top of sphere illustrating use of the ginput command

Using the graphical input (ginput) command in Matlab, as shown in Figure 3.7, the
displacement (y-coordinate) at each step could be picked off the plot and stored in vector
form. With the start location and step size of the probe known, this information could be
manipulated to form a vector corresponding to machine position at each step. Thus twenty
discrete data points (position, displacement) are known and a polynomial fit can be used to
find the maximum. The top-sphere polynomial fit is shown in Figure 3.8 and the side-
sphere fit in Figure 3.9.
Figure 3.8: Polynomial fit for top-sphere sweep

Figure 3.9: Polynomial fit for side-sphere sweep
From the polynomial fit of the top sphere data, the maximum could easily be extracted. Thus the X-position of the machine corresponding to this maximum could be determined. In a similar manner, the Z-maximum was identified from the polynomial fit of the side sphere data. These results are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Top Sphere Measurement</th>
<th>Side Sphere Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{\text{max}} = -1.5835 \text{ in.}$</td>
<td>$Z = 2.480470 \text{ in.}$</td>
</tr>
<tr>
<td>$Z_{\text{max}} = 1.6040 \text{ in.}$</td>
<td>$X = -0.707030 \text{ in.}$</td>
</tr>
</tbody>
</table>

Table 3.1: Global X and Z-positions of top and side maxima

This information allowed for the direct calculation of the femoral head radius. The absolute difference in either X or Z-coordinates above minus the probe radius yields values of femoral head radius $R$, as described by Equations 3.1 and 3.2:

$$R = |X_{\text{top}} - X_{\text{side}}| - r_{\text{probe}}$$  \hspace{1cm} (3.1)

$$R = |Z_{\text{top}} - Z_{\text{side}}| - r_{\text{probe}}$$  \hspace{1cm} (3.2)

Using Equation 3.1, the radius of the femoral head is calculated to be 0.81397 inches.

### 3.2.2 Sphericity Measurement

The radial information derived made it possible to determine seven evenly spaced test locations for surface measurement. These test positions measure the surface characteristics in the upper hemisphere of the femoral head. One additional test position is located just south of the equator. This setup is pictured in Figure 3.10, with $\phi_1$ corresponding to position 1, $\phi_2$ corresponding to position 2, and so forth. These seven probe locations are represented by the blue spheres and are equally spaced about the profile of the femoral head.
The corresponding (X, Z) coordinates of these latitudes are the global test positions of interest. This information was hard-coded into the machine and G-code was again used to control the motion of the probe and spindle. This program instructed the machine to rotate the spindle, move the probe within range of the first test position, collect data for a few revolutions, and move to the next point until data from all seven latitudes were collected. Initial testing of these global positions revealed that during rotation, a few data points were situated outside the range of the capacitance probe. The location of these positions was manually adjusted. Table 3.2 defines the global coordinates of each test position one through seven.
The data collected at each test position were isolated. Figure 3.11 depicts the
test position 3 results illustrating the actual probe reading and the
easurement taken at position 3. The blue line represents the actual probe reading and the
pink line represents the error measurement after the once-around (fundamental) component
was removed from the data set.

<table>
<thead>
<tr>
<th>Position No.</th>
<th>X</th>
<th>Z</th>
<th>$\phi$ (rad)</th>
<th>$\phi$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.711687</td>
<td>1.501057</td>
<td>-0.73079</td>
<td>-41.871</td>
</tr>
<tr>
<td>2</td>
<td>-0.705744</td>
<td>1.604000</td>
<td>-0.57253</td>
<td>-32.804</td>
</tr>
<tr>
<td>3</td>
<td>-0.724214</td>
<td>1.786108</td>
<td>-0.46999</td>
<td>-26.929</td>
</tr>
<tr>
<td>4</td>
<td>-0.786921</td>
<td>1.966325</td>
<td>-0.40459</td>
<td>-23.181</td>
</tr>
<tr>
<td>5</td>
<td>-0.895587</td>
<td>2.142945</td>
<td>-0.36286</td>
<td>-20.790</td>
</tr>
<tr>
<td>6</td>
<td>-1.075804</td>
<td>2.314617</td>
<td>-0.33968</td>
<td>-19.462</td>
</tr>
<tr>
<td>7</td>
<td>-1.370175</td>
<td>2.450004</td>
<td>-0.33355</td>
<td>-19.111</td>
</tr>
</tbody>
</table>

Table 3.2: Global (X, Z) positions and latitudes $\phi$ of seven test positions

Figure 3.11: Test position 3 results illustrating the actual probe reading and the error
measurement after removal of the fundamental component
As explained, the once-around is an artifact of the imperfect centering of the hip joint on the spindle. This periodic waveform can easily be removed from the measurement by modeling the data as a Fourier Series. The coefficients, \( A \) and \( B \), of the series were computed using Equations 3.3 and 3.4. The probe reading \( X \) is a function of the rotation angle \( \theta \), which ranges from 0 to \( 2\pi \). The variable \( N \) is the number of discrete data points.

\[
A = \frac{2}{N} \sum_{i=1}^{N} X(\theta_i) \cdot \cos(\theta_i) d\theta 
\]

(3.3)

\[
B = \frac{2}{N} \sum_{i=1}^{N} X(\theta_i) \cdot \sin(\theta_i) d\theta 
\]

(3.4)

The once-around waveform can be computed from these coefficients, as defined in Equation 3.5. To determine the actual error of the measurement, this waveform is simply subtracted from the data.

\[
X(\theta) = A \cdot \cos(\theta) + B \cdot \sin(\theta) 
\]

(3.5)

To ensure repeatability, six runs were executed. The measurements were taken with two different sampling frequencies at each of three rotational speeds. The spindle speeds tested were 10, 20, and 30 RPMs; the sampling frequencies were 512 Hz and 2.6 kHz. All six of these test runs showed very good repeatability. Figure 3.12 shows a polar plot of the error in position 3, with the enlarged image emphasizing the almost negligible variance in data between the three rotational speeds.
The measurements were also found repeatable when the sampling frequency was varied. The data sets sampled at 512 Hz are used in the remainder of this analysis as there is less noise in the data.

### 3.3 Results

The data convincingly indicate that the femoral head is very spherical in shape. However, there is a definite two-lobe form that is apparent in the exaggerated error plot of Figure 3.13. This figure pictures each of the seven error measurements plotted in the same X-Y plane. An arbitrary radius of 0.8 thousandths is added to the actual error measurement to aid in visual representation.
Figure 3.13: X-Y view of seven error profiles with an arbitrary radius of 0.8 (thousandths)

The results of this testing indicate that although the error ranges about 0.5 thousandths, the average error across all data sets (the root mean square value) is 0.1314 thousandths of an inch. This confirms the belief that the prosthetic femoral head is incredibly round. The absolute maximum error measured does not exceed 0.3864 thousandths, and the average absolute maximum error of the seven data sets is 0.2392 thousandths. These relatively large maximum errors may be a result of scratches and other imperfections in the surface finish.

The femoral head profile appears to have a two-lobe shape when a gain of 1000 is applied to the error measurement. This is apparent in Figures 3.14 and 3.16, which depict exaggerated profiles. Without a gain imposed on the error measurements, the femoral head profile looks seemingly spherical, as Figures 3.15 and 3.17 illustrate.
Figure 3.14: Femoral head, exaggerated profile

Figure 3.15: Femoral head, actual profile
Figure 3.16: X-Y view of exaggerated profile

Figure 3.17: X-Y view of actual profile
Chapter 4: Conclusions

The methods employed in this thesis were successful in determining the real form of the seemingly round prosthetic femoral head. The results of this research have shown that the error measurement is exceptionally small, and is in fact almost negligible when compared to the radius. However amplification of the error clearly shows the prosthetic femoral head has a definite two-lobe shape. Though this aspherical profile is not visible to the eye, the data are conclusive and repeatable.

In terms of future studies, valuable information could be obtained using a similar surface metrology approach to examine the surface characteristics of a worn prosthesis. The results from a study of this nature could be used in comparison with the presented data to more fully quantify how wear affects the shape of the femoral head. A comparison would give detailed information as to the limiting factors affecting the lifespan of the implant. This data could therefore be used to determine a more robust design for the prosthesis. Furthermore, studies related to the shape of the prosthetic acetabular cup could provide another means of comparison. In combination, this data has the potential to redefine the standard in hip joint prosthetics, extending the lifetime of these devices and thereby lessening the need for total joint replacement surgeries altogether.
Appendix A: Femoral Head Preprocessing Program

A.1 Matlab Preprocessing Code

% Femoral Head Metrology Testing
% Preprocessor - Run #1
% 3/26/08
% RPM20_FS512.mat taken by Liz and Eddy
% units are Volts with 1 V/mil probes

clear all; clf; clc;

load RPM20_FS512.mat

% Define sensitivity of probes
sens = 1; % V/mil

% Change units from volts to milli-inch
encoder = (VCAP_DATA(:,1) - mean(VCAP_DATA(:,1)))/sens;
probe = VCAP_DATA(:,2)/sens;

% Low-pass filter the capacitance probe data
[b,a] = yulewalk(8, [0 .1 .15 1], [1 1 0 0]);
probe = filtfilt(b, a, probe);

% Define time
time = 1/VCAP_SAMPLERATE * [0: length(probe)-1]';
clear a b VCAP_DATA

% find encoder points
i = find(encoder <= 0.05);
encoder(i) = 0;
i = find(encoder > 0.05);
encoder(i) = 1;
j = find(diff(i) == 1);
encoder(i(j)+1) = 0;
i = find(encoder > 0.5);

% Find first point in each position
for inc = 1: 7,
    clf
    plot(time,encoder,time,probe)
    title('RPM = 20, F_s = 512 Hz')
xlabel('Time (sec)')
ylabel('Displacement (milli-inch)')
legend('Encoder','Probe')
pt = ginput(1);
set(gca, 'XLim', [pt(1) pt(1)+20])
pt = ginput(1);
enctick = round(VCAP_SAMPLERATE*pt(1));
[enctick, j] = min(abs(i-enctick));

A(inc) = i(j);
B(inc) = i(j+3);
end
clear VCAP_SAMPLERATE ans encoder enctick i j pt sens inc

save RPM20_PRE.mat
Appendix B: Femoral Head Analysis

B.1 Matlab Analysis I Code

% Femoral Head Metrology Testing
% Data Analysis I
% Test for repeatability
% 4/7/08

%clear all; clf; clc;
z = [1.501057 1.604 1.786108 1.966325 2.142945 2.314617 2.450004];

%% ---------- RPM 10 Data ---------- %
load RPM10_PRE.mat
enc = round(mean((B-A))/3);
data = zeros(enc, 7);
for inc = 1: 7,
    data(:, inc) = mean([probe(A(inc) + (1: enc))'; probe(A(inc) + enc + (1: enc))'; probe(A(inc) + 2*enc + (1: enc))']);
end
for inc = 1: 7,
a = 2/length(data) * sum(data(:, inc).*cos(2*pi*[0: length(data)-1]'/length(data)));
b = 2/length(data) * sum(data(:, inc).*sin(2*pi*[0: length(data)-1]'/length(data)));
data(:, inc) = data(:, inc) - a*cos(2*pi*[0: length(data)-1]'/length(data)) - b*sin(2*pi*[0: length(data)-1]'/length(data));
end
y = detrend(data, 'constant');
phi = 2*pi*(0: enc-1)'/enc;
Z = ones(enc,1)*z;
for i = 1:7;
    [X(:,i),Y(:,i)] = pol2cart(phi,(y(:,i))+0.8);
end

clear a b A B inc data enc probe time

for i = 1:7;
    figure(i)
polar(phi,y(:,i)+1,'g')
title('Position Error')
hold on
end
figure(8)
plot3(X,Y,Z,'g')
title('Femoral Head Error Measurement')
xlabel('X Position Error (thousandths)')
ylabel('Y Position Error (thousandths)')
zlabel('Z Position (in)')
hold on

clear X Y Z

%% ----------- RPM 20 Data --------------- %%

load RPM20_PRE.mat
enc = round(mean((B-A))/3);
data = zeros(enc, 7);

for inc = 1: 7,
    data(:, inc) = mean([probe(A(inc) + (1: enc))'; probe(A(inc) + enc + (1: enc))'; probe(A(inc) + 2*enc + (1: enc))'])';
end
Y20 = data;

for inc = 1: 7,
a = 2/length(data) * sum(data(:, inc).*cos(2*pi*[0: length(data)-1]'/length(data)));
b = 2/length(data) * sum(data(:, inc).*sin(2*pi*[0: length(data)-1]'/length(data)));
data(:, inc) = data(:, inc) - a*cos(2*pi*[0: length(data)-1]'/length(data)) - b*sin(2*pi*[0: length(data)-1]'/length(data));
end

y = detrend(data, 'constant');
phi = 2*pi*(0: enc-1)/enc;
Z = ones(enc,1)*z;
for i = 1:7;
    [X(:,i),Y(:,i)] = pol2cart(phi,(y(:,i))+0.8);
end

clear a b A B inc enc

for i = 1:7;
    figure(i)
polar(phi,y(:,i)+1,'m')
title('Position Error')
hold on
end

figure(8)
plot3(X,Y,Z,'m')
hold on
figure(9)
plot(phi,Y20(:,3), phi,y(:,3),'m')
title('Position 3 Surface Measurement')
xlabel('Rotation Angle (rad)')
ylabel('Displacement (thousandths)')
axis([0 2*pi -8 4])

clear X Y Z Y20

%%% ---------- RPM 30 Data ---------- %%%

load RPM30_PRE.mat
enc = round(mean((B-A))/3);
data = zeros(enc, 7);
for inc = 1: 7,
data(:, inc) = mean([probe(A(inc) + (1: enc))'; probe(A(inc) + enc + (1: enc))'; probe(A(inc) + 2*enc + (1: enc))']));
end
for inc = 1: 7,
a = 2/length(data) * sum(data(:, inc).*cos(2*pi*[0: length(data)-1]/length(data)));
b = 2/length(data) * sum(data(:, inc).*sin(2*pi*[0: length(data)-1]/length(data)));
data(:, inc) = data(:, inc) - a*cos(2*pi*[0: length(data)-1]/length(data)) - b*sin(2*pi*[0: length(data)-1]/length(data));
end
y = detrend(data, 'constant');
phi = 2*pi*(0: enc-1)/enc;
Z = ones(enc,1)*z;
for i = 1:7;
    [X(:,i),Y(:,i)] = pol2cart(phi,(y(:,i))+0.8);
end

clear a b A B inc data enc probe time
for i = 1:7;
    figure(i)
polar(phi,y(:,i)+1,'b')
title('Position Error')
end

figure(8)
plot3(X,Y,Z,'b')
axis equal
axis([-1.25 1.25 -1.25 1.25 1.5 2.5])

clear z i X Y Z
B.2 Matlab Analysis II Code

```matlab
% Femoral Head Metrology Testing
% Data Analysis II
% Mesh plots of surface
% 4/7/08

clear all; clf; clc;

load RPM20_PRE.mat
enc = round(mean((B-A))/3);
data = zeros(enc, 7);
for inc = 1: 7,
data(:, inc) = mean([probe(A(inc) + (1: enc))'; probe(A(inc) + enc + (1: enc))'; probe(A(inc) + 2*enc + (1: enc))']);
end

for inc = 1: 7,
a = 2/length(data) * sum(data(:, inc).*cos(2*pi*[0: length(data)-1]'/length(data)));
b = 2/length(data) * sum(data(:, inc).*sin(2*pi*[0: length(data)-1]'/length(data)));
data(:, inc) = data(:, inc) - a*cos(2*pi*[0: length(data)-1]'/length(data)) - b*sin(2*pi*[0: length(data)-1]'/length(data));
end

y = detrend(data, 'constant');

phi = 2*pi*(0: enc-1)/enc;

clear a b inc data

%% -------------------- Surface Mesh -------------------- %%%

theta = [-0.117534986 0 0.20883942 0.426880945 0.6645673 0.950442086 1.323789502];

Theta = ones(enc,1)*theta;                % account for probe angle
probe = y./cos(Theta-(pi/4));

R = 0.81397;                                 % inches
gainR = R + probe/10;                        % with 1000 fold gain
realR = R + probe*10^-3;                     % no gain

for i = 1:7;
    % Define surface coords. with gain
    x(:,i) = gainR(:,i)*cos(theta(i));
    Z(:,i) = gainR(:,i)*sin(theta(i));
    [X(:,i),Y(:,i)] = pol2cart(phi,x(:,i));
```
% Define real surface coords.
rx(:,i) = realR(:,i)*cos(theta(i));
rZ(:,i) = realR(:,i)*sin(theta(i));
[rX(:,i), rY(:,i)] = pol2cart(phi, rx(:,i));
end

figure(1)
colormap(jet(200))
surf(X, Y, Z, y)
camlight left; lighting phong, shading interp
caxis([-1 .3])
axis equal
title({'Femoral Head Sphericity'; '\itwith Gain'})
xlabel('X Position Error')
ylabel('Y Position Error')
zlabel('Z Position')

figure(2)
colormap(jet)
figure(2)
surf(rX, rY, rZ, y)
camlight left; lighting phong, shading interp
axis equal
title({'Femoral Head Sphericity'; '\itwithout Gain'})
xlabel('X Position Error (in)')
ylabel('Y Position Error (in)')
zlabel('Z Position (in)')
clear R Theta X Y Z gainR i phi probe rX rY rZ realR rx theta x y
References


