Correcting capacitive displacement measurements in metrology applications with cylindrical artifacts
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Abstract
Metrology applications commonly require non-contact, capacitive sensors for displacement measurements due to their nanometer resolution. In some metrology applications, for example, the measurement of roundness and spindle error motion, the displacements of stationary and rotating cylindrical artifacts are measured. Error from using a conventionally calibrated sensor with a non-flat (e.g., cylindrical) target is typically neglected, but these errors cannot be ignored for nanometer-level accuracy. The capacitance between a sensor and a cylindrical target is less than that of a sensor with a flat target, which causes four effects. As the diameter of the target shrinks, the sensitivity of the sensor increases, the sensing range decreases, the sensing range shifts towards the target, and the nonlinearity increases. These errors can be greatly reduced by either calibrating sensors with the correct target surface or by determining corrections for post-processing data. This paper quantifies and experimentally verifies these errors for a commonly used sensor, and a simulation of a nanometer-level measurement of out-of-roundness and spindle error motion demonstrates that measurement accuracy is improved with corrected sensitivities.

Keywords: Displacement sensors; Cylinder; Artifact; Metrology

1. Introduction
Many metrology applications require non-contact measurements with nanoscale resolution. Capacitive displacement sensors are a common solution to this measurement problem. A typical realization of the capacitive sensor is a sensing electrode and guard ring enclosed within a grounded sensor body. Fig. 1 shows this configuration in a commercial sensor used in precision applications. The sensor is positioned perpendicular to a target electrode formed by a conductive surface in a workpiece, machine, or instrument. In most instruments and in fine motion control, the target is usually a flat surface with characteristic dimensions significantly larger than the sensing electrode, which can be as small as 1 mm. The working distance and sensing range vary with the sensor size and electronic gain, but typical values for working distance are from 100 to 1000 μm with a typical sensing range of 50–2000 μm.

It is standard practice in applications such as measuring the roundness of a cylindrical shaft [1,2], measuring the accuracy of spindle rotation [3,4], and measuring the accuracy of machine tools [5,6], to use capacitive sensors that target spherical and cylindrical artifacts. However, capacitive displacement sensors are typically calibrated targeting flat surfaces. Measurement error, which is often ignored, is introduced when such a calibrated sensor is used with a non-flat target or artifact. The magnitude of the error depends upon the geometry of the capacitive sensor, the diameter of the cylindrical target, the working distance, and possibly the sensor’s electronics. In some cases the error is negligibly small, but we find that it is significant when interested in measuring nanometer displacements.

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The capacitance is proportional to the permittivity of free space $\varepsilon_0$, the relative permittivity of the dielectric material separating the sensor and target $\varepsilon_r$, and the area of the sensing electrode $A$. The capacitance is inversely proportional to the gap $g$ that separates the sensor and target, since the electric field strength decreases as the gap is increased. The permittivity of free space is a constant approximately $8.854 \times 10^{-12}$ F/m, and the relative permittivity, which is always greater than one, is the ratio of the dielectric’s permittivity to the permittivity of a vacuum.

$$C = \frac{\varepsilon_0 \varepsilon_r A}{g}$$  \hspace{1cm} (1)

In practice, capacitive displacement sensors use finite sized electrodes, which results in fringing of the electric field around the perimeter of the electrodes. This non-uniform electric field causes deviations from the ideal capacitance given by Eq. (1). However, the homogeneity of the electric field is greatly improved by incorporating a guard ring into the sensor, as suggested by Lord Kelvin. A guard ring is an annular electrode that encloses the sensing electrode; the two electrodes are separated by an insulator but operated at the same electric potential. Maxwell [17] found that the increase in capacitance due to the field fringing into the annulus between the guard ring and sensing area is as small as a few parts per million when the radial separation $w$ between the sensing electrode and guard ring is small compared to the gap. In such cases, the capacitance between the sensing electrode and target may be estimated with Eq. (1).

Two approaches are used to detect displacements with changes in capacitance. In the first approach, the sensor’s capacitance varies linearly when displacements alter the overlapping area $A$ of the electrodes. The measured displacement is tangential to the electrode surfaces. Sensors using this ‘shearing’ approach [18–20] are most suitable for measuring larger displacements with less sensitivity. The second approach is more common in precision metrology and used by the sensor in Fig. 1 since it is suited for higher sensitivity but smaller displacements [11,12,21]. In this approach, the area is unchanged and displacements are normal to the electrode surfaces, producing changes in the gap $\Delta g$ between the electrodes. A linear (rather than hyperbolic) dependence on $g$ is obtained by considering the inverse of the capacitance $C^{-1}$ as shown in Eq. (2) for an ideal parallel-plate capacitor.

$$C^{-1} = \frac{g}{\varepsilon_0 \varepsilon_r A}$$  \hspace{1cm} (2)

Electronics that detect small changes in capacitance often use AC bridges [22] such as the transformer ratio bridge [10,11,13,23,24] or other circuits [25–28]. Most of these circuits compare the impedance, which is proportional to $C^{-1}$, of the sensing electrode relative to ground with the impedance of a fixed, internal reference capacitance $C_{ref}$. The impedances of the two capacitances are equal when $g$ equals the nominal gap distance $g_{nom}$. The electronics produce an output voltage $V$ that is proportional by gain $G$ to the difference between the impedances so that $V$ is zero when $g$ equals
This suggests the linear sensing law in Eq. (4), where the output \( V \) is proportional to changes in the gap \( \Delta g \) that are measured from the nominal gap \( g_{\text{nom}} \), and the proportionality constant is the sensor’s sensitivity \( S \). The methods described in this paper are independent of the sensor’s electronics, as long as they produce an output voltage of this type.

\[
V = S \Delta g \quad (3)
\]

\[
V = G(C^{-1} - C_{\text{ref}}^{-1}) \quad (4)
\]

In practice, the actual capacitance usually differs from ideal values such as the one given in Eq. (1) or those given for other electrode geometries by Heerens and Vermeulen [29] or Heerens [30,31]. Discrepancies result from complex electrode geometries, physical factors, or electrical factors. Electrical factors can include stray capacitance [27], temperature drift [32], and dynamic hysteresis [33]. Hicks and Atherton [34] and several other researchers considered physical factors that necessitate adjustments to the capacitance values:

- finite width of the annular guard ring [10,35];
- thermal expansion of electrodes [11];
- fringing of the electric field between the sensing electrode and guard ring [12,17,35,36];
- variation in the relative permittivity due to temperature and pressure fluctuations [32];
- nonflatness or noncoplanarity of the electrodes [34,35];
- nonparallelism (tilt) of the sensor’s face and target surface [33,39,37,38];
- roughness or damage to the sensing electrode, guard ring, or target surfaces [39];
- thickness of the electrodes [39];
- elastic deformation due to electrostatic forces [40].

Computational techniques such as finite element analyses (FEA) enable the capacitance to be determined for complex electrode geometries or deviations due to the physical factors listed above. Several researchers used FEA to determine the capacitance as a function of the gap distance [41–44], and Lányi [45] used two-dimensional FEA analyses to study the effects of surface irregularities and machining errors on parallel plate capacitive sensors.

Because of the variety of factors that affect the sensing law given in Eq. (3), commercially available sensors are individually calibrated to determine their sensitivity \( S \) and their nonlinearity. In nearly all cases, the calibration is conducted by recording the voltage \( V \) output by the sensor’s electronics while the sensor targets a flat surface in air. Displacements of the sensor from the nominal gap are compared with another precision sensor such as a displacement measuring interferometer [36], a traceable linear scale, or a Fabry-Perot etalon [46].

Table 1 lists the calibrated sensitivities for the representative sensor shown in Fig. 1 when it targets a flat surface in air. This sensor operates in two alternative modes. The first mode yields a larger sensing range (50.8 \( \mu m \)) but lower sensitivity (-0.394 V/\( \mu m \)), and the second yields a high sensitivity (-1.969 V/\( \mu m \)) but smaller sensing range (10.0 \( \mu m \)). Both modes produce an analog voltage that ranges from -10 to +10 V. This sensor has excellent linearity when targeting a flat surface; calibration data indicates that the sensor is linear within 0.33% over the full measurement range for its high sensitivity setting.

3. Determination of capacitance with finite element analyses

It is necessary to know how the capacitance varies with changes in the gap to quantitatively predict the effects of a cylindrical target. Unfortunately, closed-form analytical solutions, such as the ideal relation in Eq. (1) or the more complex relations described by others [29–31,34], can seldom account for the complexities of common sensors. For instance, at least four factors with the representative sensor will contribute to discrepancies from an ideal capacitance. First, analytical solutions generally assume that the target surface is flat. Second, the radial distance separating the sensing electrode and guard ring is not small compared to the nominal gap. Third, analytical solutions usually assume that air separates the guard ring and sensing electrode rather than an epoxy insulator (\( e_s = 3.8 \)). Kahn et al. [36] previously found that this affected sensor output. Finally, the guard ring has a tapered geometry so that it is not entirely flat.

To account for these factors, we use electrostatic finite element analyses to determine how the capacitance varies with changes in the gap. Electrostatic FEA can determine the voltage drop across dielectric materials, the electric field, and the lump capacitances between electrodes, and it readily accommodates different material properties for the dielectrics, multiple conductive electrodes, and complex geometries. In this work, as with our previous work on spherical targets [7,8], we find that the FEA results correlate well with experimental validations.

Table 1: Nominal gaps, sensitivities, and nonlinearity for low and high sensitivities of the representative capacitive sensor

<table>
<thead>
<tr>
<th>Sensing range (( \mu m ))</th>
<th>Nominal gap, ( g_{\text{nom}} ) (( \mu m ))</th>
<th>Sensitivity, ( S ) (V/( \mu m ))</th>
<th>Nonlinearity (%FS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low sensitivity</td>
<td>76.2–127.0</td>
<td>101.6</td>
<td>-0.394</td>
</tr>
<tr>
<td>High sensitivity</td>
<td>20.3–38.5</td>
<td>25.4</td>
<td>-1.969</td>
</tr>
</tbody>
</table>
3.1. The finite element model

As illustrated in Fig. 2, the geometry of the sensor targeting a cylindrical artifact is reduced to one-quarter of the complete 3D geometry since two planes of symmetry exist that intersect at the centerline of the sensor. Only a portion of the cylinder’s length is modeled since the cylinder is significantly longer than the radius of the sensor. The model includes four conductors, two insulators, and the air gap. The four conductors are the electrode, the guard ring, the outer body of the sensor, and the target surface. The two insulators are the epoxy between the sensing electrode and the guard ring and the epoxy between the guard ring and the sensor’s body.

Only the non-conductive dielectric materials are meshed into finite elements since these are the only regions where an electric field is present. The conductive electrodes cannot contain electric fields, so their surfaces are represented with electric potential boundary conditions. The non-conductive regions are meshed with tetrahedral elements that are assigned appropriate values of relative permittivity. The epoxy has a relative permittivity of 3.8, and the air between the sensor and target has a relative permittivity of approximately 1.0. Each tetrahedral element contains 10 nodes, and each node has a single degree of freedom, electric potential measured in volts. Our previous spherical models [8] used infinite elements at the perimeter of the air gap to prevent electric field lines from being ‘drawn’ to the edge of the model. However, a recent comparison of results both with and without these infinite elements found identical capacitances to seven significant figures (due to concentration of field within the guard ring). Therefore, these infinite elements are not included in the present work.

Two examples in Figs. 3 and 4 illustrate typical results from an electrostatic FEA. Fig. 3 shows a plot of the electric potential within the air and epoxy. The sensing electrode and guard ring are both set at a 5 V potential, while the sensor’s body and the cylindrical target are set to 0 V (ground). It is observed from this plot that the guard ring concentrates the electric field within the confines of the guard ring since the contours extend little beyond the guard ring. Most of the bending of the contours and fringing of the field occurs...
beyond the sensing electrode, where it has less effect on the detected capacitance between the sensing electrode and target surface.

The quiver plot in Fig. 4 illustrates the direction and magnitude of the electric field within the dielectric materials for the potential distribution shown in Fig. 3. The electric field vectors are calculated from the gradient of the electric potential, and point toward the surface of lowest potential. The electric field is strongest where the sensing electrode and the target are in closest proximity, and the strength decreases as the distance between the surfaces increases.

3.2. Capacitance as a function of gap and target diameter

Although graphical plots such as those shown in Figs. 3 and 4 provide physical insight, they do not provide a lumped capacitance, which is necessary for quantitatively predicting the effects of cylindrical targets. In cases that include more than two conductors, a matrix of lumped capacitances between each pair of conductors must be determined. Fig. 5 shows the four conductors and the six capacitances between each pair of conductors. To determine these capacitance values, the FEA must be solved multiple times, with different voltages applied to different electrodes. Fortunately, the ANSYS commercial software provides a convenient macro titled CMATRIX that solves the energy stored in the electric field under various boundary conditions and computes the capacitances automatically. Although this macro yields all of the lumped capacitance values, it is only the capacitance $C_{14}$ between conductors 1 and 4 (sensing electrode and target) that is detected by the sensor’s electronics.

The complete FEA procedure based on the CMATRIX macro is illustrated in Fig. 6. An outer loop generates the model geometry and meshes the insulators for each gap distance, so that the end result is a list of capacitance values as a function of gap distance $g$. Between 12 and 15 increments of the gap are analyzed for each cylindrical diameter; the increments are $5.10^{-6}$ m for the low sensitivity and $1.02^{-6}$ m for the high sensitivity. Capacitance values are calculated for cylindrical targets with diameters of 6.35, 9.53, 12.70, 15.88, 19.05, 22.23, and 25.40 mm. For comparison, the capacitances are also determined for a flat target. The values of capacitance for the low sensitivity range between 0.217 and 0.526 pF, while the capacitances for the high sensitivity range between 0.593 and 1.822 pF.

4. Effects on displacement sensing

Analysis of the lumped capacitances between the sensing electrode and target $C_{14}$ reveals four effects from using cylindrical targets. As the diameter of the target is reduced, the sensing range decreases and the nominal gap moves toward the target. More significantly, both the sensitivity and nonlinearity increase, which is important since it degrades the accuracy of metrology as demonstrated in Section 5. All of these effects are more pronounced when sensors operate at their highest sensitivity and the diameter of the cylindrical target is small.
4.1. Reduction and shift of the sensing range

Figs. 7 and 8 give the inverse of the capacitance between the sensing electrode and target (1/C1) as a function of the gap distance g for both sensitivities and the seven target diameters. The ideal relation in Eq. (2) suggests that the inverse capacitance should be a straight line with positive slope, and this is very nearly the case for the representative sensor. Vertical lines in Figs. 7 and 8 indicate the minimum, nominal, and maximum gaps for a flat target (data in Table 1). Tracing these lines to the y-axis gives the inverse capacitances that correspond to the minimum, nominal, and maximum gaps for the case of a flat target.

The sensing range for each cylinder diameter is determined by the intersections of the horizontal lines (for minimum, nominal, and maximum capacitance values) with the cylindrical data curves. The technique is illustrated for the Ø6.35 mm target at low sensitivity in Fig. 7 and at high sensitivity in Fig. 8. On the low sensitivity graph, the nominal gap distance shifts from around 102 μm for the flat target to around 68 μm for the Ø6.35 mm target. The sensing range reduces from about 51 μm to about 43 μm. On the high sensitivity graph, the nominal gap distance shifts from 25 μm for the flat target to around 10 μm for the Ø6.35 mm target. The sensing range for the high sensitivity decreases from around 10 μm for the flat surface to around 6 μm. Tables 2 and 3 list the shift of the gap toward the target and the reduced sensing ranges for the remaining target diameters.

4.2. Increases in sensitivity and nonlinearity

To quantify increases in the sensitivity and nonlinearity due to cylindrical targets, the inverse capacitances plotted in Figs. 7 and 8 are converted to output voltages. The relationship presented in Eq. (4) indicates that the output voltage V should be linearly proportional to the change in gap Δg with respect to the nominal gaps that are listed in Tables 2 and 3.

The gain G of the sensor’s electronics is determined from the slope of the voltage-inverse capacitance line for the flat target, as shown in Eq. (6). The sensor electronics provide a ±10 V signal as the inverse capacitance varies between the minimum and maximum values for a flat target. The inverse capacitance for the low sensitivity setting ranges between 2.347 and 3.663 pF⁻¹, and the high sensitivity setting ranges between 0.394 and 1.969 V/μm. The nonlinearity is determined from the slope of the voltage-inverse capacitance line for the flat target, as shown in Eq. (6).
between 0.763 and 1.078 pF$^{-1}$. Therefore, the gains are determined to be $G = -15.2$ V pF and $G = -63.5$ V pF for the low and high sensitivities, respectively.

$$ G = \frac{V_{\text{max}} - V_{\text{min}}}{\left(\frac{1}{C_{\text{14}}}\right)_{\text{min}} - \left(\frac{1}{C_{\text{14}}}\right)_{\text{max}}} $$

Eq. (7) gives the output voltage by multiplying the gain of the electronics with the difference between the inverse capacitance $1/C_{14}$ and nominal inverse capacitance $(1/C_{14})_{\text{nom}}$.

The nominal inverse capacitances for the low and high sensitivities were 0.332 and 1.083 pF$^{-1}$, respectively, as shown in Figs. 7 and 8.

$$ V = G \left(\frac{1}{C_{\text{14}}} - \left(\frac{1}{C_{\text{14}}}\right)_{\text{nom}}\right) $$

(7)

Figs. 9 and 10 show how the output voltage $V$ varies as a function of $\Delta g$ for the low and high sensitivities, respectively. The slope of these lines, which is the sensitivity $\Delta g$ from Eq. (4), clearly increases as the diameter of the target cylinder decreases. Corrected sensitivities for each diameter of the target are determined by finding the slope of the least-squares line through the data points in the ±10 V range for each target cylinder. These results are listed in Table 4. As the diameter of the cylindrical target increases, the sensitivities approach

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**Table 2**
Predicted changes in the gap and sensing range for low sensitivity ($-0.394$ V/μm)

<table>
<thead>
<tr>
<th>Target diameter (mm)</th>
<th>Gap minimum (μm)</th>
<th>Nominal (μm)</th>
<th>Maximum (μm)</th>
<th>Change in nominal gap (%)</th>
<th>Sensing range (μm)</th>
<th>Change in range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.35</td>
<td>47.2</td>
<td>68.3</td>
<td>90.5</td>
<td>-33</td>
<td>43.3</td>
<td>-15</td>
</tr>
<tr>
<td>9.53</td>
<td>54.2</td>
<td>76.8</td>
<td>100.2</td>
<td>-24</td>
<td>46.0</td>
<td>-9</td>
</tr>
<tr>
<td>12.70</td>
<td>58.6</td>
<td>81.9</td>
<td>105.7</td>
<td>-19</td>
<td>47.1</td>
<td>-7</td>
</tr>
<tr>
<td>15.88</td>
<td>61.5</td>
<td>85.3</td>
<td>109.7</td>
<td>-16</td>
<td>48.2</td>
<td>-5</td>
</tr>
<tr>
<td>19.05</td>
<td>65.6</td>
<td>87.6</td>
<td>112.3</td>
<td>-14</td>
<td>48.7</td>
<td>-4</td>
</tr>
<tr>
<td>22.23</td>
<td>65.2</td>
<td>89.4</td>
<td>114.3</td>
<td>-12</td>
<td>49.1</td>
<td>-3</td>
</tr>
<tr>
<td>25.40</td>
<td>66.4</td>
<td>90.7</td>
<td>115.8</td>
<td>-11</td>
<td>49.4</td>
<td>-3</td>
</tr>
<tr>
<td>Flat</td>
<td>76.2</td>
<td>101.6</td>
<td>127.0</td>
<td>-61</td>
<td>6.4</td>
<td>-41</td>
</tr>
</tbody>
</table>

**Table 3**
Predicted changes in the gap and sensing range for high sensitivity ($-1.969$ V/μm)

<table>
<thead>
<tr>
<th>Target diameter (mm)</th>
<th>Minimum (μm)</th>
<th>Gap nominal (μm)</th>
<th>Maximum (μm)</th>
<th>Change in nominal gap (%)</th>
<th>Sensing range (μm)</th>
<th>Change in range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.35</td>
<td>7.1</td>
<td>10.0</td>
<td>13.1</td>
<td>-41</td>
<td>6.0</td>
<td>-41</td>
</tr>
<tr>
<td>9.53</td>
<td>9.2</td>
<td>12.7</td>
<td>16.4</td>
<td>-50</td>
<td>7.2</td>
<td>-29</td>
</tr>
<tr>
<td>12.70</td>
<td>10.7</td>
<td>14.6</td>
<td>18.6</td>
<td>-42</td>
<td>7.9</td>
<td>-22</td>
</tr>
<tr>
<td>15.88</td>
<td>11.9</td>
<td>16.1</td>
<td>20.3</td>
<td>-37</td>
<td>8.4</td>
<td>-18</td>
</tr>
<tr>
<td>19.05</td>
<td>12.9</td>
<td>17.1</td>
<td>21.6</td>
<td>-32</td>
<td>8.7</td>
<td>-15</td>
</tr>
<tr>
<td>22.23</td>
<td>13.6</td>
<td>18.0</td>
<td>22.6</td>
<td>-29</td>
<td>9.0</td>
<td>-12</td>
</tr>
<tr>
<td>25.40</td>
<td>14.2</td>
<td>18.8</td>
<td>23.4</td>
<td>-26</td>
<td>9.2</td>
<td>-10</td>
</tr>
<tr>
<td>Flat</td>
<td>20.3</td>
<td>25.4</td>
<td>30.5</td>
<td>-61</td>
<td>10.2</td>
<td>-41</td>
</tr>
</tbody>
</table>

**Fig. 9.** Output voltage for low sensitivity ($-0.395$ V/μm) as functions of the change in gap and target diameters (predicted by FEA).
those for a flat target (−0.394 and −1.969 V/μm). For the Ø6.35 mm target and low sensitivity, the percent change in sensitivity is about 67%. The sensitivity for the less severe case of a Ø25.4 mm target still differs from the flat reference by about 3 and 11% for the low and high sensitivities, respectively.

Increases in the sensor’s nonlinearity are observed by computing the residuals of the least-squares line. Figs. 11 and 12 show the residuals computed for the low and high sensitivities for each target diameter. The worst-case residual voltage of about 0.6 V or 3% is observed for the 6.35 mm target and high sensitivity, which is about an order or magnitude larger than the nonlinearity for a flat surface.

With the sensitivity increases comes pronounced reduction of linearity. In addition, the sensing range may have shifted so close to the workpiece that it becomes impractical to position the sensor at the right nominal gap. This was the case for the experimental data for the smallest cylinders and highest sensitivity in Table 4.

4.3. Validation of corrected sensitivities

Experimental validation of the corrected sensitivities determined by FEA was conducted with the setup shown in Fig. 13. One capacitive displacement sensor targets a flat surface, while a second sensor targets one of a set of cylindrical targets with various diameters. Both sensors are mounted collinearly in a common bracket in accordance with the Abbe Principle to minimize the effects of off-axis motion. The flat and cylindrical targets are on opposing faces of the same block. The targets are mounted on the moveable base of a Moore No. 3 Universal Measuring Machine that translates in the direction along the sensors’ axes. A translation produces equal changes in the gap distances (but opposite in sign). If both capacitive sensors had identical sensitivities, the sum of the measured outputs would be zero and independent of the position of the target stage. Because pairs of data points from each sensor are collected simultaneously, both sensors measure fluctuations in the stage displacement equally. The output from the sensor that targets the flat surface determines

Table 4
Comparison of corrected sensitivities determined by finite element analyses and experimentally

<table>
<thead>
<tr>
<th>Target (mm)</th>
<th>Nominal sensitivity (V/μm)</th>
<th>-0.394</th>
<th>-1.969</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEA (V/μm)</td>
<td>Exp (V/μm)</td>
<td>Difference (%)</td>
</tr>
<tr>
<td>Flat</td>
<td>-0.394</td>
<td>-0.395</td>
<td>0.2</td>
</tr>
<tr>
<td>25.40</td>
<td>-0.406</td>
<td>-0.407</td>
<td>0.2</td>
</tr>
<tr>
<td>22.23</td>
<td>-0.409</td>
<td>-0.410</td>
<td>0.2</td>
</tr>
<tr>
<td>19.05</td>
<td>-0.411</td>
<td>-0.412</td>
<td>0.2</td>
</tr>
<tr>
<td>15.88</td>
<td>-0.415</td>
<td>-0.419</td>
<td>0.9</td>
</tr>
<tr>
<td>12.70</td>
<td>-0.421</td>
<td>-0.425</td>
<td>0.9</td>
</tr>
<tr>
<td>9.53</td>
<td>-0.432</td>
<td>-0.437</td>
<td>1.2</td>
</tr>
<tr>
<td>6.35</td>
<td>-0.460</td>
<td>-0.463</td>
<td>0.6</td>
</tr>
</tbody>
</table>
During calibration with two flat target surfaces the sensor output repeats within 20 nm and matches the manufacturer’s calibration data within 0.25%, which is small compared to the effects observed due to target curvature. This result confirms the accuracy of the sensor alignment, and the insensitivity of the experimental hardware to vibration, noise, and thermal drift.

Prior to testing, the sensor is centered by finding the ‘high spot’ on the cylindrical target. By checking the repeatability of the comparative sensor output with different amounts of deliberate decentering, it was found that the effect is negligible for sensor centering within 50 μm of the high spot. It is straightforward to find the high spot to within 25 μm or better using the three axes of the Moore measuring machine.
thereby satisfying this requirement. Once the highest point on each cylindrical target is found, both capacitive sensors are adjusted axially in the sensor bracket such that each sensor is within a half micrometer of the middle of its sensing range.

As predicted by the finite element analysis, the sensor targeting the cylindrical surface had greater sensitivity than the ‘flat-sensing’ sensor, and showed increased nonlinearity. The experimental results listed in Table 4 show excellent agreement with the finite element models. The maximum discrepancy between model and experiment was less than 1.2% for the $-0.394 \text{ V/\mu m}$ sensitivity and less than about 7.7% for the $-1.969 \text{ V/\mu m}$ sensitivity.

5. Corrections in roundness and spindle metrology

One of the most common uses of capacitive sensors with cylindrical targets is found in precision spindle and roundness metrology. Fig. 14 shows a typical measurement with a lapped cylindrical target such as a gage pin mounted on the rotor of a precision spindle. One or more sensors target the rotating cylinder and measure the runout of the cylindrical surface. In high precision applications, the form error of the gage pin is significant with respect to the radial error motion of the spindle. The literature documents several methods of accurately separating the artifact form error from the spindle’s radial error motion [9,47–49]. In fact, the best air bearing spindles actually have less radial error motion (less than 10 nm) than most lapped cylindrical artifacts.

As shown in the previous sections, the radius of the cylindrical target affects capacitive sensor measurements, but using corrected sensitivities such as those listed in Table 4 for the representative sensor minimizes this effect. However, additional errors result since the cylindrical artifact is inevitably not perfectly centered on the axis of rotation, and this eccentricity causes the rotating target to explore the nonlinear response of the capacitive sensor. Simply employing corrected sensitivities does not eliminate error due to the nonlinear residuals.

5. Corrections in roundness and spindle metrology

One of the most common uses of capacitive sensors with cylindrical targets is found in precision spindle and roundness metrology. Fig. 14 shows a typical measurement with a lapped cylindrical target such as a gage pin mounted on the rotor of a precision spindle. One or more sensors target the rotating cylinder and measure the runout of the cylindrical surface. In high precision applications, the form error of the gage pin is significant with respect to the radial error motion of the spindle. The literature documents several methods of accurately separating the artifact form error from the spindle’s radial error motion [9,47–49]. In fact, the best air bearing spindles actually have less radial error motion (less than 10 nm) than most lapped cylindrical artifacts.

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6. Conclusion

Commercial capacitive displacement sensors are often calibrated with flat surfaces, even though some precision manufacturing and metrology applications require displacement measurements of cylindrical surfaces. This common occurrence leads to four detrimental effects that become more severe as the diameter of the target reduces. First, the sensitivity of the sensing system becomes larger than the calibrated sensitivity, resulting in overstatement of measured displacements. Second, the sensing range decreases, and third, the sensing range shifts towards the target surface. Lastly, the relationship between the output voltage and the actual displacement, though highly linear for flat targets, becomes increasingly nonlinear. The nonlinearity becomes even more important when a cylindrical artifact is eccentric with respect to an axis of rotation.

Electrostatic finite element analyses or experimental techniques like that used for verification successfully predict corrected sensitivities and nonlinear residuals. Higher gain settings, by virtue of their shorter working distances,
are prone to the largest errors in gain and linearity. For the representative sensor, the FEA and experiments indicate that the error in sensitivity can be as much as 67% with significantly more nonlinearity.

These concerns are addressable in demanding metrology applications, either by calibrating sensors with a cylindrical target or by using corrected sensor sensitivities. For all diameter cylinders, a corrected sensitivity compensates for most of the measurement error. However, for the smallest cylinders and the higher sensitivities, nonlinear terms might also be included in the correction to achieve even more accurate displacement measurements.

References


